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Connections between Abstract Quantum Theory and Space-Time Structure. III. Vacuum Structure and Black Holes

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Quantum-theoretic considerations for the ground state of a black hole result in a change of its interior solution. It is shown that the interior of a Schwarzschild black hole can be modeled by an ur-theoretically described Robertson-Walker space-time. Thereby the Schwarzschild singularity is changed into a Friedman singularity.

1. INTRODUCTION

One expects that an appropriate unification of quantum theory and gravitation theory should lead to an explanation of the observed smallness of the cosmological constant and to a better understanding of the space-time singularities of classical general relativity. We do not think that one should try to avoid or even remove space-time singularities in quantized gravity; we rather take singularities as precious hints to look for a new type of unification. The usual attempts to construct a union of quantum theory and gravity are applications of quantization procedures to gravitation theory retaining the space-time continuum even at very small distances.

In this paper we do not presuppose a space-time continuum first but start with abstract quantum theory, i.e., the quantum theory of binary alternatives (Drieschner *et al.*, 1987; Görnitz, 1988*a*,*b*). Space-time is introduced via the invariance group of the "ur," the quantized binary alternative. This invariance group turns out to be $U(2)$.

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The time development is given by the subgroup $U(1)$ and for the symmetric space representing position space we have

$$U(2)/U(1) = SU(2) = S^3$$

[for details see Görnitz (1988*a*)]. Taking this as a model for cosmic space where the evolution of this cosmos is described by a growing number of urs, we get a Robertson–Walker space-time (with $k = +1$) with the right order of an effective cosmological constant being a consequence of this model (Görnitz, 1988*b*).

Now let us consider a black hole. Assigning an entropy to the black hole (Bekenstein, 1973, 1974) led to the problem that one must also assign a temperature, i.e., the black hole had to radiate, which is absolutely forbidden by the classical theory. The resolution of this difficulty was given by Hawking (1975), showing that quantum effects cause black holes to create and emit particles, i.e., application of quantum theory led to reasonable results for the exterior region of the black hole.

Here we want to employ quantum-theoretic considerations for the interior solution. But how can we model an interior black hole solution taking quantum effects into account?

The central point for quantum theory is the existence of a ground state and its dependence on the system's extension. The horizon of a black hole defines an informationally closed volume, i.e., a finite volume where no information about its internal states can be obtained outside. The ground state of such an ideal box has to depend on the radius of the event horizon. Another informationally closed volume is a Robertson–Walker universe with $k = +1$. We show now that the interior of a Schwarzschild black hole can be modeled by an ur-theoretically described Robertson–Walker space-time.

2. THE ENERGY-MOMENTUM TENSOR

We assume that the quantized binary alternatives, the urs, behave like a perfect fluid with energy-momentum tensor

$$_{(\mathrm{ur})}T_{ab} = (\rho + p)u_a u_b - p g_{ab} \tag{1}$$

where ρ is the energy density density of the ur-fluid, u_a the normalized four-velocity vector, and the pressure is given by

$$p = -\rho/3 \tag{2}$$

A derivation of this relation between energy density and pressure is given in Görnitz (1988*a*,*b*).

We take $\rho \geq 0$, i.e., the pressure becomes negative. Usually one does not like negative pressures in physics. Nevertheless, a negative pressure is not unknown; it may occur, for example, in certain nonequilibrium states. In general relativity, the pressure contributes to the gravitational force, which in the case of negative pressure leads to the effects of a repulsive contribution to the gravitational force. Actually, some models which try to solve the horizon problem of the early universe use a negative pressure

$$p_{\text{vac}} = -\rho_{\text{vac}}$$

associated with the cosmological constant as the energy density of the vacuum to derive inflation (see, e.g., Guth 1981; Albrecht and Steinhard, 1982; Linde, 1982; Weinberg, 1988).

We want to show now that our negative pressure of equation (2) stays within the limits set by all relevant energy conditions in general relativity.

The *weak* energy condition states that the energy as measured by any observer is nonnegative. Every physically reasonable energy-momentum tensor is diagonalizable. Written in an orthonormal basis, the three principal pressures p coincide for uniform pressure and the eigenvalue ρ represents the rest energy density. In this case, the weak energy condition is equivalent to

$$\rho \geq 0 \qquad \text{and} \qquad \rho + p \geq 0 \tag{3}$$

The *strong* energy condition which limits the stresses and guarantees the existence of singularities is equivalent to

$$\rho + 3p \geq 0 \qquad \text{and} \qquad \rho + p \geq 0 \tag{4}$$

Finally, the *dominant* energy condition, which states that the velocity of the energy flow is always less than the speed of light, is equivalent to

$$\rho \geq |p| \tag{5}$$

As is easily seen, (2) fulfils all these energy conditions. It is interesting to notice that this negative pressure is the maximal negative pressure to obey the strong energy condition and is somehow just the opposite of radiation with

$$p = \rho/3$$

The energy-momentum density of the vacuum acts like a cosmological constant. Let us therefore decompose ${}_{(\text{ur})}T_{ab}$ into a sum of energy-momentum tensors for matter, radiation, and vacuum:

$${}_{(\text{ur})}T_{ab} = {}_{(\text{mat})}T_{ab} + {}_{(\text{rad})}T_{ab} + {}_{(\text{vac})}T_{ab} \tag{6}$$

where

$$
{}_{(\mathrm{mat})}T_{ab} = \begin{bmatrix} \rho_{\mathrm{mat}} & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{bmatrix} \tag{7}
$$

$$
{}_{(\mathrm{rad})}T_{ab} = \begin{bmatrix} \rho_{\mathrm{rad}} & & & \\ & -\rho_{\mathrm{rad}}/3 & & \\ & & -\rho_{\mathrm{rad}}/3 & \\ & & & -\rho_{\mathrm{rad}}/3 \end{bmatrix} \tag{8}
$$

$$
{}_{(\mathrm{vac})}T_{ab} = \begin{bmatrix} \lambda & & & \\ & \lambda & & \\ & & \lambda & \\ & & & \lambda \end{bmatrix} \tag{9}
$$

3. THE METRIC

We want to construct now an interior Schwarzschild solution with energy momentum tensor ${}_{(\mathrm{ur})}T_{ab}$ as given by (1) and (2).

We start with the most general form of a spherically symmetric metric

$$
ds^2 = e^{2A}\, dt^2 - e^{2B}\, dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{10}
$$

where

$$
A = A(r, t), \qquad B = B(r, t)
$$

By Birkhoff's theorem, a spherically symmetric vacuum solution has to be static, i.e., has to be the Schwarzschild solution. Therefore, we have to choose

$$
A(r, t) = \tfrac{1}{2}\ln(1 - R_s/r) \tag{11a}
$$

$$
B(r, t) = -\tfrac{1}{2}\ln(1 - R_s/r) \tag{11b}
$$

with R_s as a constant, which leads to the Schwarzschild metric

$$
\begin{aligned} ds^2 &= (1 - R_s/r)\, dt^2 - [(1 - R_s/r)^{-1}\, dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)] \\ &\text{for } R_s < r < \infty \end{aligned} \tag{12}
$$

For the interior solution we take

$$
A(r, t) = 0 \tag{13a}
$$

$$
B(r, t) = -\tfrac{1}{2}\ln(1 - r^2/R_s^2) \tag{13b}
$$

i.e., the line element

$$ds^2 = dt^2 - [(1 - r^2/R_s^2)^{-1}\, dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)] \quad \text{for } 0 < r < R_s \tag{14}$$

Einstein's field equations

$$G_{ab} = -\kappa T_{ab} \tag{15}$$

with the energy-momentum tensor of a perfect fluid lead to

$$G_0^0 = -\kappa\rho, \qquad G_\beta^\alpha = \kappa p \qquad (\alpha, \beta = 1, 2, 3) \tag{16}$$

and with (14) to

$$\rho = 3/(\kappa R_s^2), \qquad p = -1/(\kappa R_s^2) \tag{17}$$

The total mass is computed as

$$2M = \kappa \int_0^{R_s} \rho r^2\, dr = R_s \tag{18}$$

i.e., R_s is actually the Schwarzschild radius.

To join the interior and exterior solutions together, one normally demands that the kinetic pressure is zero on the surface of the fluid ball. Actually, a match could not be achieved for an interior of uniform and nonzero positive pressure and uniform energy density, because the infinite surface pressure gradient would blow off the outer layers of the fluid and send a rarefaction wave propagating inward, thereby destroying the uniform distribution.

In our model the uniform pressure is negative and not zero on the surface. But in this case it results from contributions of the energy density of the vacuum, which acts like an effective cosmological constant, i.e., for

$${}_{\text{eff}}\Lambda = -\lambda \tag{19}$$

equations (6)-(9) yield

$${}_{\text{eff}}G_{ab} + {}_{\text{eff}}\Lambda g_{ab} = -\kappa({}_{(\text{mat})}T_{ab} + {}_{(\text{rad})}T_{ab}) \tag{20}$$

If we apply the coordinate transformation

$$r' = r/R_s \tag{21}$$

to the line element (14), we get

$$ds^2 = dt^2 - R_s^2[(1 - r'^2)^{-1}\, dr'^2 + r'^2(d\theta^2 + \sin^2\theta\, d\phi^2)] \tag{22}$$

a stationary Robertson-Walker solution, i.e., an Einstein cosmos with curvature radius R_s.

The stationarity of this solution is closely connected with requirement (2) for ${}_{(\mathrm{ur})}T_{ab}$. Taking a general Robertson–Walker metric $(k=+1)$

$$ds^2 = dt^2 - a(t)^2[(1-r^2)^{-1}\,dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)] \tag{23}$$

the field equations with the energy-momentum tensor of a perfect fluid give

$$\kappa(\rho+3p) = -6\ddot{a}(t)/a(t) \tag{24}$$

This implies

$$\ddot{a}(t)=0 \qquad \text{iff} \quad p=-\rho/3 \tag{25}$$

i.e.,

$$a(t)=a(0)+vt \tag{26}$$

with v as constant expansion velocity.

4. DISCUSSION

The standard Schwarzschild solution has an implicit supposition that the formation of a horizon has no influence on the structure of the vacuum. This is a natural assumption in the framework of general relativity. But from a quantum-theoretic point of view, the formation of a horizon (like any enclosure) should show a back reaction on the quantum ground state. In our model we take this into consideration via the introduction of a negative pressure associated with the vacuum energy density. With this assumption, the interior of a Schwarzschild black hole can be described by a complete stationary Robertson–Walker space-time, i.e., replacing the Schwarzschild singularity by a Friedmann singularity.

As Penrose (1982) conjectured, principles other than the ones we use already in physics should come into play at a singularity, bringing along with them the time-asymmetric character necessary to explain the second law of thermodynamics. In our model, time asymmetry is actually built in. It is one of the fundamental postulates of abstract quantum theory, and therefore for the theory of quantized binary alternatives. We take this as a hint that the so-called singularity problem may in fact be a problem of understanding the concept of time.

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